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A Variational Approach for Solving an Inverse Vibration Problem

Leonardo D. Chiwiacowsky and Haroldo F. de Campos Velho
Laboratório Associado de Computação Aplicada - LAC
Instituto Nacional de Pesquisas Espaciais - INPE
São José dos Campos, SP, Brazil
[leodc,haroldo]@lac.inpe.br

Paolo Gasbarri
Dipartimento di Ingegneria Aerospaziale e Astronautica
Università degli Studi di Roma, La Sapienza
Rome, Italy
paolo.gasbarri@uniroma1.it

Abstract. The present investigation is focused on the solution of a dynamic inverse problem which is concerned with the assessment of damage in structures by means of measured vibration data. This inverse problem has been presented as a optimization problem and has been solved through the use of the iterative regularization method, i.e. the Conjugate Gradient method. The results have been presented in a satisfactory form when a small structure with few degrees-of-freedom (DOF) is considered, however when a higher DOF-structure is considered, the simple application of the iterative regularization method is not more satisfactory, being necessary the application of an additional methodology. To solve this difficulty, in this paper a new approach, based on the use of the Genetic Algorithm (GA) method has been proposed. The GA method is used to generate a primary solution which is employed as the initial guess for the conjugate gradient method. The application of this new approach has been showed that better results can be achieved, although the computational time for the application here analyzed could be increase. The damage estimation has been evaluated using noiseless and noisy synthetic experimental data, and the reported results are concerned with a truss and a beam-like structures both modeled through a finite element technique. Moreover, in order to take into account the reduced set of experimental data to be employed in the optimization algorithm, a Guyan reduction technique has been adopted on the finite element formulation.

INTRODUCTION

Considerable research and effort over the last few decades has taken place in the field of system identification problem, for different reasons. One of the most interesting applications involves the monitoring of structural integrity through the identification of damage. It is well known that damage modifies the dynamic response of a structure and, at the same time, that changes in its behavior may be associated with the decay of the system's mechanical properties [1].

The damage identification problem is displayed as an inverse vibration problem, since the damage evaluation is achieved through the determination of the stiffness coefficient variation, or the stiffness coefficient by itself. The inverse problem solution is generally unstable, therefore, small perturbations in the input data, like random errors inherent to the measurements used in the analysis, can cause large oscillations on the solution. In general the inverse problem, i.e. the ill-posed problem, is presented as a well-posed functional form, whose solution is obtained through an optimization procedure.

Based on these considerations, various papers have examined the use of measured variations in dynamic behavior to detect structural damage. A variety of experimental, numerical and analytical techniques has already been proposed to solve the damage identification problem, and have received notable attention due to its practical applications [2]. These methods are usually classified under several categories, such as frequency and time domain methods, parametric and non-parametric models, deterministic and stochastic approaches [3, 4].

Among the classical methods, recently the use of the conjugate gradient method with the adjoint equation [5, 6], or Variational Approach, which has been used successfully in thermal sciences, has also been presented as a satisfactory choice to face the damage identification problem. Some papers regarding to the use of the Alifanov's method in inverse vibration problems can be found in the literature, for instance, Huang [7, 8] has been estimated the time-dependent stiffness coefficients considering spring-mass systems with one and multiple degrees of freedom. Also, Castello and Rochinha [9] have been identified the elastic and damping parameters of a bar-like structure using the Alifanov's method. On the other hand, among the non-classical methods the stochastic methods, represented by the GA method, represent a powerful choice to face non trivial optimization problems.

GAs are search algorithms based on the mechanics of nature selection and natural genetics [10], which are design to efficiently search large, nonlinear, discrete and poorly search space where expert knowledge is scarce or difficult to model and where classical optimization techniques fail. Some papers regarding to the use of the GA method alone can be found in the literature, for instance Barbosa and Borges [11] have been identified damage scenarios in a framed structure, while Mares and Surace [12] have been used the GA method for the simultaneous location and quantification of damage in a truss and a beam structures.

It has been noticed that when the system considered presents a slightly high number of DOFs, the conjugate gradient method becomes sensible in relation to some parameters, such as the initial guess. By virtue of the above considerations, the simple application of the iterative regularization method could be not more satisfactory, being necessary the application of an additional methodology. In this work, a hybrid approach is proposed to solve the inverse structural vibration problem for the damage identification, which can be estimated through the determination of some stiffness parameters of the structure. The genetic algorithm is applied initially in order to determine a better initial guess for a following application of the standard conjugate gradient method. This hybrid approach has already been used for an inverse damage problem considering spring-mass systems and it has produced good results [13, 14] and in this work more complicated structures will be studied and also the effect of the noisy in the experimental data has been considered.

THE INVERSE ANALYSIS

The inverse vibration problem of estimation of the stiffness coefficients has been considered in this work. The unknown stiffness coefficients have been recovered from the synthetic system displacement measurements of a forced dynamical system with M-DOF. The inverse analysis with the conjugate gradient method involves the following steps [5, 6]:

(i) the solution of direct problem;

(ii) the solution of sensitivity problem;

(iii) the solution of adjoint problem and the gradient equation;

(iv) the conjugate gradient method of minimization;

(v) the stopping criteria;

(vi) the solution algorithm.

Next, a brief description of basic procedures involved in each of these steps is presented.

The Direct Problem

The M-DOF dumped systems considered in this work are presented in the Figs. 1-2 and the mathematical formulation of this forced vibration systems is given by

$$\mathbf{M}\,\ddot{\mathbf{x}}(t) + \mathbf{C}(t)\,\dot{\mathbf{x}}(t) + \mathbf{K}(t)\,\mathbf{x}(t) = \mathbf{f}(t)\,, \quad (1)$$

with initial conditions

$$\mathbf{x}(0) = \mathbf{x}_0 \quad \text{and} \quad \dot{\mathbf{x}}(0) = \dot{\mathbf{x}}_0\,. \quad (2)$$

Here $\mathbf{M}$ represents the system mass matrix, $\mathbf{K}(t)$ the time-dependent stiffness matrix, $\mathbf{C}(t)$ the time-dependent damping matrix, $\mathbf{f}(t)$ the external forces vector, and $\mathbf{x}(t)$ the displacements vector. There exists no analytical solution for Eq. (1) for any arbitrary functions of $\mathbf{K}(t)$, $\mathbf{C}(t)$, and $\mathbf{f}(t)$, obtained through the finite element method. For this reason, the numerical solution with the *Newmark* method [15], will be applied to solve Eq. (1). The direct problem calculates the system displacement $\mathbf{x}(t)$, if initial conditions, system parameters $\mathbf{M}$, $\mathbf{K}(t)$ and $\mathbf{C}(t)$, and the time-dependent external forces $\mathbf{f}(t)$ are known.

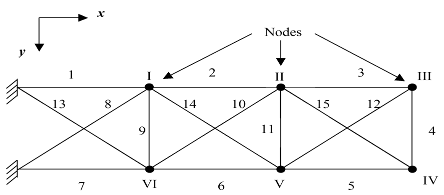

Figure 1: The three-bay truss structure considered in this work.

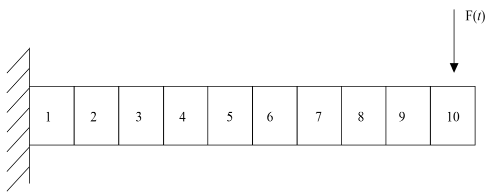

Figure 2: The 10-DOF beam structure considered in this work.

The Sensitivity Problem

The problem involves N unknown time-dependent stiffness parameters, which constitute the elements of the stiffness matrix $\mathbf{K}(t) = f[\boldsymbol{\mathcal{K}}(t)]$, where $\boldsymbol{\mathcal{K}}(t) = [K_1(t), ..., K_N(t)]$ and the parameters $K_i(t)$, $i = 1, ..., N$ represent the structural stiffness parameters of the finite element; for instance for a bar-like structure $K_i = EA/L_e$, for a beam-like structure $K_i = EI/L_e^3$, where E is the Young's module, I is the inertial moment, A is the cross section area and L_e is the length of the finite element. In order to derive the sensitivity problem for each unknown function $K_i(t)$, each unknown stiffness parameter should be perturbed at a time. Supposing that the $K_i(t)$ is perturbed by a small amount $\triangle K_i(t)\,\delta(i-j)$, where the $\delta(\cdot)$ is the Dirac-delta function and $j = 1,\ldots,N$, it results in a small change in displacements by the amounts $\triangle x_{ij}(t)$. The sensitivity problem is obtained by replacing in the direct problem, Eqs. (1)-(2), $K_i(t)$ by $K_i(t) + \triangle K_i(t)\,\delta(i-j)$, $x_i(t)$ by $x_i(t)+\triangle x_{ij}(t)$, and by subtracting from the resulting expression the original direct problem, and also by neglecting the second-order terms. Therefore, N sensitivity problems have been obtained, since $j = 1,\ldots,N$, i.e., a different sensitivity problem for each perturbed stiffness parameter. The sensitivity problem is defined by the following system of differential equations

$$\mathbf{M}\,\triangle\ddot{\mathbf{x}}_j(t)+\mathbf{C}(t)\,\triangle\dot{\mathbf{x}}_j(t)+\mathbf{K}(t)\,\triangle\mathbf{x}_j(t) = \triangle\mathbf{K}_j(t)\,\mathbf{x}(t)\,, \tag{3}$$

where $j = 1,\ldots,N$ and with initial conditions

$$\triangle\mathbf{x}_j(0) = 0 \quad \text{and} \quad \triangle\dot{\mathbf{x}}_j(0) = 0\,. \tag{4}$$

The Adjoint Problem and the Gradient Equation

In general the inverse problem does not satisfy the requirements of existence and uniqueness, then it must be solved as an optimization problem requiring that the unknown function $\boldsymbol{\mathcal{K}}(t)$ should minimize the functional vector $J[\boldsymbol{\mathcal{K}}(t)]$ defined by

$$\boldsymbol{J}[\boldsymbol{\mathcal{K}}(t)] = \int_0^{t_f} [\,\mathbf{x}(t) - \mathbf{x}^{exp}(t)]^{\mathrm{T}}\,[\,\mathbf{x}(t) - \mathbf{x}^{exp}(t)]\;dt, \tag{5}$$

where t_f is the final time, $\mathbf{x}(t)$ and $\mathbf{x}^{exp}(t)$ are the computed and measured displacements at time t, respectively. The adjoint problem is developed by multiplying Eq. (1) by the *Lagrange* multiplier vector $\boldsymbol{\lambda}(t)$, integrating the resulting expression over time domain and then adding this result to the functional given by Eq. (5). The resulting expression is given by

$$\begin{aligned}\boldsymbol{J}[\boldsymbol{\mathcal{K}}] \;&=\; \int_0^{t_f} [\,\mathbf{x} - \mathbf{x}^{exp}]^{\mathrm{T}}\,[\,\mathbf{x} - \mathbf{x}^{exp}]\;dt\,+\\ &+\int_0^{t_f} \boldsymbol{\lambda}^{\mathrm{T}}\,\{\mathbf{M}\,\ddot{\mathbf{x}} + \mathbf{C}\,\dot{\mathbf{x}} + \mathbf{K}\,\mathbf{x} - \mathbf{f}\}\;dt\,.\end{aligned} \tag{6}$$

The variation $\triangle\boldsymbol{J}_j[\boldsymbol{\mathcal{K}}(t)]$ of the functional is obtained by perturbing $\mathbf{K}(t)$ by $\triangle\mathbf{K}_j(t)$ and $\mathbf{x}(t)$ by $\triangle\mathbf{x}_j(t)$ in Eq. (6), and subtracting from it the original Eq. (6). Neglecting the second-order terms, the resulting expression is given by

$$\begin{aligned}\triangle\boldsymbol{J}_j[\boldsymbol{\mathcal{K}}] \;&=\; \int_0^{t_f} 2\,[\,\mathbf{x} - \mathbf{x}^{exp}]^{\mathrm{T}}\,\triangle\mathbf{x}_j\;dt\,+\\ &+\int_0^{t_f} \boldsymbol{\lambda}^{\mathrm{T}}\left\{\mathbf{M}\,\triangle\ddot{\mathbf{x}}_j + \mathbf{C}\,\triangle\dot{\mathbf{x}}_j + \mathbf{K}\,\triangle\mathbf{x}_j + \triangle\mathbf{K}_j\,\mathbf{x}\right\}\;dt.\end{aligned} \tag{7}$$

When the second term of the right-hand side of this expression is integrated by parts and the null initial conditions of the sensitivity problem are employed, the following adjoint problem is obtained for the determination of the *Lagrange* multiplier vector $\boldsymbol{\lambda}(t)$. Since the adjoint problem has no dependence on the perturbed stiffness ($\triangle\mathbf{K}_j$), the subscript j has been neglected and the adjoint problem is defined by the expression

$$\mathbf{M}\,\ddot{\boldsymbol{\lambda}}(t)-\mathbf{C}(t)\,\dot{\boldsymbol{\lambda}}(t)+\mathbf{K}(t)\,\boldsymbol{\lambda}(t) = 2\,[\mathbf{x}^{exp}(t) - \mathbf{x}(t)]\,, \tag{8}$$

with final conditions

$$\boldsymbol{\lambda}(t_f) = 0 \quad \text{and} \quad \dot{\boldsymbol{\lambda}}(t_f) = 0\,. \tag{9}$$

The adjoint problem is different from the standard initial value problems because a final time condition at time $t = t_f$ is specified instead of the classical initial condition at $t = 0$. However, the problem (8) can be transformed to an initial value problem by introducing a new time variable: $\tau = t_f - t$. Then the standard technique of *Newmark* method can be applied for the solution of the transformed problem.

During the process for obtaining the adjoint problem, the following integral term was used

$$\triangle\boldsymbol{J}[\boldsymbol{\mathcal{K}}] = \int_0^{t_f} \boldsymbol{\lambda}^{\mathrm{T}}\,\triangle\mathbf{K}\,\mathbf{x}\;dt\,. \tag{10}$$

In this work the inverse vibration problem of stiffness estimation has been solved as a parameter estimation problem where the stiffness parameters have been assumed constants; i.e. $\mathbf{K}(t)$ =const., during the time. Therefore, the integral term left can be written as

$$\triangle\boldsymbol{J}[\boldsymbol{\mathcal{K}}] = \sum_{j=1}^{N}\int_0^{t_f} \boldsymbol{\lambda}^{\mathrm{T}}\,\triangle\mathbf{K}_j\,\mathbf{x}\;dt\,. \tag{11}$$

By definition, the directional derivative of $\boldsymbol{J}[\boldsymbol{\mathcal{K}}]$ in the direction of a vector $\triangle\mathbf{K}$ is given by [6]

$$\triangle\boldsymbol{J}[\boldsymbol{\mathcal{K}}] = \sum_{j=1}^{N} \boldsymbol{J}'_j\,\triangle\mathbf{K}_j\,, \tag{12}$$

where $\boldsymbol{J}'$ is the gradient direction of the functional $\boldsymbol{J}$. A comparison of Eqs. (11) and (12) reveals that the j^{th} component of the gradient direction, $\boldsymbol{J}'_j$, is given by

$$\boldsymbol{J}'_j[\boldsymbol{\mathcal{K}}] = \int_0^{t_f} \boldsymbol{\lambda}^{\mathrm{T}}\,\triangle\tilde{\mathbf{K}}_j\,\mathbf{x}\;dt\,, \tag{13}$$

where $\triangle\tilde{\mathbf{K}}_j$ refers to the j^{th} perturbed stiffness matrix, i.e. $\triangle\tilde{\mathbf{K}}_j = \partial[\triangle\mathbf{K}]/\partial K_j(t)$.

The Conjugate Gradient Method of Minimization

The iterative procedure based on the conjugate gradient method is used for the estimation of the unknown stiffness parameters $\boldsymbol{\mathcal{K}}$ given in the form [5, 6]:

$$\boldsymbol{\mathcal{K}}^{n+1} = \boldsymbol{\mathcal{K}}^n - \boldsymbol{\beta}^n\,\mathbf{P}^n, \quad n = 0, 1, 2, ..., \tag{14}$$

where $\boldsymbol{\beta}^n$ is the step size vector and $\mathbf{P}^n$ is the direction of descent vector at the step n defined as [5]

$$\mathbf{P}^n = \boldsymbol{J'}^n + \boldsymbol{\gamma}^n\,\mathbf{P}^{n-1}, \quad \text{with} \quad \gamma^0 = 0, \tag{15}$$

where $\boldsymbol{\gamma}^n$ is the conjugate coefficient vector. Note that Eq. (14) is a iterative procedure for which the stopping criteria will be discussed in next section. It should be noticed that the special case $\boldsymbol{\gamma}^n = 0$, for any n, corresponds to the steepest descent method. Different definitions of the conjugate coefficient $\boldsymbol{\gamma}^n$ are reported in the literature [5, 16]. In the present work the conjugate coefficient vector has been adopted as

$$\boldsymbol{\gamma}^n = \frac{[J'^{\,n}(\boldsymbol{\mathcal{K}})]^2}{[J'^{\,n-1}(\boldsymbol{\mathcal{K}})]^2}, \quad n = 1, 2, ... \tag{16}$$

The step size vector $\boldsymbol{\beta}^n$, appearing in Eq. (14), is determined by minimizing the functional $\boldsymbol{J}[\boldsymbol{\mathcal{K}}^{\,n+1}]$ given by Eq. (5) with respect β^n, i.e.

$$\min_{\boldsymbol{\beta}^n} J[\boldsymbol{\mathcal{K}}^{n+1}(t)] = \min_{\boldsymbol{\beta}^n} \int_0^{t_f} [\mathbf{x}(\boldsymbol{\mathcal{K}}^n - \boldsymbol{\beta}^n\,\mathbf{P}^n) - \mathbf{x}^{exp}]^2 \; dt. \tag{17}$$

By performing a *Taylor*-series expansion of the integrand of Eq. (17) the value of $\boldsymbol{\beta}^n$ for the minimum can be evaluated analiticaly:

$$\begin{aligned} \boldsymbol{\beta}^n &= \left\{ \int_0^{t_f} [\triangle\mathbf{x}(t)]^{\mathrm{T}}\,[\triangle\mathbf{x}(t)] \right\}^{-1} \times \\ &\times \left\{ \int_0^{t_f} [\triangle\mathbf{x}(t)]^{\mathrm{T}}\,[\mathbf{x}(t) - \mathbf{x}^{exp}(t)] \right\} \; dt\,. \end{aligned} \tag{18}$$

The Discrepancy Principle for the Stopping Criteria

In the absence of measurement errors, due to the experimental devices, one can use the customary stopping criteria

$$J[\boldsymbol{\mathcal{K}}] < \epsilon^*, \tag{19}$$

where $J[\boldsymbol{\mathcal{K}}]$ is defined by Eq. (5) and ϵ^* is a small specified number. However, in practical applications, measurement errors are always present; therefore the *Discrepancy Principle* [17, 5] as described below should be used to establish the stopping criteria.

Let the standard deviation σ of the measurement errors be the same for all sensors and measurements, that is,

$$\mathbf{x}(t) - \mathbf{x}^{exp}(t) \cong \sigma. \tag{20}$$

Introducing this result into Eq. (5), we obtain

$$\int_0^{t_f} \sigma^2 \; dt = \sigma^2\, t_f \equiv \epsilon^2. \tag{21}$$

Then the discrepancy principle for the stopping criteria is taken as

$$J[\boldsymbol{\mathcal{K}}] < \epsilon^2. \tag{22}$$

The Solution Algorithm

The standard computational procedure of the conjugate gradient method is summarized in the following algorithm:

Step 1: Choose an initial guess $\boldsymbol{\mathcal{K}}^0$.

Step 2: Solve the direct problem, Eqs. (1)-(2), to obtain $\mathbf{x}(t)$.

Step 3: Solve the adjoint problem, Eqs. (8)-(9), to obtain the *Lagrange* multiplier vector $\boldsymbol{\lambda}(t)$.

Step 4: Knowing $\boldsymbol{\lambda}(t)$, compute the gradient function vector $\boldsymbol{J}'(\boldsymbol{\mathcal{K}})$ from Eq. (13).

Step 5: Compute the conjugate coefficient vector $\boldsymbol{\gamma}^n$ from Eq. (16).

Step 6: Compute the direction of descent vector $\mathbf{P}^n$ from Eq. (15).

Step 7: Setting $\triangle\mathbf{K} = \mathbf{P^n}$ [6], solve the sensitivity problem, Eqs. (3)-(4), to obtain $\triangle\mathbf{x}(t)$.

Step 8: Compute the step size vector $\boldsymbol{\beta}^n$ from Eq. (18).

Step 9: Compute $\boldsymbol{\mathcal{K}}^{n+1}$ from Eq. (14).

Step 10: Test if the stopping criteria, Eq. (22), is satisfied. If not, go to step 2.

As mentioned before, the inverse vibration problem of damage identification, considered in this work, have already been solved through the use of the iterative regularization method, i.e. the Conjugate Gradient method [7, 8, 9]. The results have been presented in a satisfactory form when both lumped-parameter and bar-like forced systems have been considered using a small number of unknown stiffness parameters. However, it has been observed by the authors that when a slightly higher number of unknown parameters is considered, the application of the iterative regularization method, in the standard form is not more satisfactory, being necessary the application of an additional methodology [13, 14].

It has also been noticed that the initial guess choice becomes more decisive when more stiffness parameters are sought to be estimated. For avoiding this difficulty, it has been proposed a new approach where the GA method is used to generate a primary solution which is employed as the initial guess for the conjugate gradient method. This new approach could be inserted in the above procedure as the new **Step 1**. The stochastic approach has already been used in an inverse heat conduction problem, as unique methodology, and has produced good results [3], and also this proposed hybrid approach has already been used in an inverse vibration problem where a 10-DOF lumped parameter system has been considered [13].

The Stochastic Method – Genetic Algorithm

Genetic algorithms are essentially optimization algorithms whose solutions evolve somehow from the science of genetics and the processes of natural selection - the Darwinian principle. They differ from more conventional optimization techniques since they work on whole populations of encoded solutions, and each possible solution is encoded as a gene.

The most important phases in standard GAs are selection (competition), reproduction, mutation and fitness evaluation. Selection is an operation used to decide which individuals to use for reproduction and mutation in order to produce new search points. Reproduction or crossover is the process by which the genetic material from two parent individuals is combined to obtain one or more offsprings. Mutation is normally applied to one individual in order to produce a new version of it where some of the original genetic material has been randomly changed. Fitness evaluation is the step in which the quality of an individual is assessed [10].

The application of GA method to solve the problem of damage identification is also a minimization problem, as well as the gradient conjugate method. The same functional form, or fitness function, given by Eq. (5), is employed

$$J[\mathcal{K}] = \int_0^{t_f} \left[\mathbf{x}(t) - \mathbf{x}^{exp}(t) \right]^{\mathrm{T}} \left[\mathbf{x}(t) - \mathbf{x}^{exp}(t) \right] \, dt \,,$$

where $\mathbf{x}^{exp}(t)$ is the experimental system displacement vector and $\mathbf{x}(t)$ is the system displacement vector obtained obtained by Eqs. (1)-(2) using updating values for the stiffness matrix.

In this GA implementation, the algorithm operates on a fixed-sized population which is randomly generated initially. The members of this population are fixed-length and real-valued strings that encode the variable which the algorithm is trying to optimize ($\mathcal{K}$). Next, the evolutionary operators employed are presented: Tournament Selection [18], Geometrical Crossover [19], Non-uniform Mutation [19], Epidemical Operator [3].

REDUCED MODEL

A properly comparison of the real structural model and the finite element model is possible when the completeness of the experimental data is available. However, the number of measured DOFs use to be smaller than the number of DOFs which describe the complete finite element model. Usually, this problem is faced applying a reduction technique: the original system is reduced to a system whose new dimension is equal to the number of measured DOFs, identified as principal DOFs. In this work two different forced systems have been considered: a 3-bay truss structure and a 20-DOF beam structure. For the second one the Guyan Reduction technique [20] have been applied in order to reduce the complete finite element model, of dimension n, to a reduced model which presents a dependence on the measurable vertical displacements only. In the Guyan reduction method the relationship between the principal (p) and secondary (s) DOFs is established using the static analysis, where it is assumed that no external loads are applied to the secondary DOFs. The relationship between the complete and the principal measured DOFs is expressed as

$$\mathbf{x} = \mathbf{T_g} \, \mathbf{x_p} \,, \tag{23}$$

where $\mathbf{T_g}$ is the $n \times p$ Guyan reduction matrix, where the relevant expression is not reported here for the sake of brevity. Now, the problems identified by the Eqs. (1)-(2), (3)-(4) and (8)-(9) have been reduced and show an explicit dependency only by the measured DOFs (the vertical displacements of the beam). The new reduced system matrices $\mathbf{M_r}$, $\mathbf{C_r}$ and $\mathbf{K_r}$ are computed as following

$$\begin{aligned} \mathbf{M_r} &= \mathbf{T_g}^{\mathrm{T}} \mathbf{M} \, \mathbf{T_g} \,; \\ \mathbf{C_r} &= \mathbf{T_g}^{\mathrm{T}} \mathbf{C} \, \mathbf{T_g} \,; \\ \mathbf{K_r} &= \mathbf{T_g}^{\mathrm{T}} \mathbf{K} \, \mathbf{T_g} \,. \end{aligned} \tag{24}$$

RESULTS AND DISCUSSIONS

In this work it has been presented an alternative hybrid approach to solve the damage identification problem involving the estimation of the unknown stiffness parameters. In order to evaluate the capability of this new approach two different structures have been considered, both modeled through the finite element technique: a 3-bay truss structure (12-DOF) obtained from a finite element model of bar and a 20-DOF beam structure. The real experimental data, to be used in the optimization method, have been simulated by using the same finite element model. In order to generate a damaged structure a reduced value on some stiffness parameters have been imposed on the finite element discretization.

It has been noticed that when the considered systems present a low number of DOFs the standard conjugate gradient method is applied in a satisfactory way to the damage identification problem [14]. However when this systems present a slightly high number of DOFs, the standard conjugate gradient method could not be applied directly to solve the associated inverse vibration problem, because the initial guess of the unknown quantities can not be chosen arbitrarily. For this reason, it has been adopted the stochastic method of GA to find a primary solution which has been assumed as initial guess.

Regarding to the genetic algorithm method, the minimization problem is defined by equation (5) and the parameters employed in this technique are taken as: tournament selection operator; geometrical crossover operator; non-uniform mutation operator; mutation probability of 50%; fixed population size of 100 individuals, fixed maximal generation number of 2000 and 20% best individuals kept when the epidemical operator is activated.

The Truss Structure

The numerical example considered here is a 3-bay truss structure (see Fig. 1), composed of 15 bars, clamped at one end. The truss is composed by aluminum bars ($\rho = 2700\ kg/m^3$ and $E = 70\ GPa$) with a square cross section area $A = 9.0 \times 10^{-4}\ m^2$, where the nondiagonal elements are $1.0\ m$ long. For this numerical example it has been used the finite element method to calculate the mass and the stiffness matrices that appear in Eq. (1), note that for this example one finite element for each bar has been used. As far as the damping matrix is concerned, it has been assumed that it is proportional to the undamaged stiffness matrix $\mathbf{C} = 10^{-5}\,\mathbf{K}$. Furthermore, it has been assumed an external force of intensity $\mathbf{f}(t) = 1000.0$ N applied at nodes III and IV in the positive diagonal direction constant with time and the following initial conditions have been adopted $\mathbf{x}(0) = 0$ and $\dot{\mathbf{x}}(0) = 0$. The following damage configuration has been considered: a 30% damage over the element 7; 20% over the element 13; 15% over the element 2; 10% over the element 10 and a 5% damage over the element 4. All the others elements have been assumed as undamaged for the generation of the experimental data.

The numerical results have been obtained considering two different cases: noiseless and noisy experimental data; i.e the displacements of the nodes (see Fig. 1) of the structure along x and y directions. Numerical simulations have been performed assuming the final time as $t_f = 3$ s and a time step $\triangle t = 3.0 \times 10^{-4}$ s.

The experimental data have been obtained from the exact solution of the direct problem (noiseless data) or by adding a random perturbation error to the exact solution of the direct problem (noisy data) in the following form

$$\mathbf{x}^{exp}(t) = \mathbf{x}(t) + \sigma\,\mathcal{R}, \qquad (25)$$

where σ is the standard deviation of the errors and $\mathcal{R}$ is a random variable taken from a normal distribution such that $\mathcal{R} \sim \mathcal{N}$ormal(0;1). For numerical purposes, the σ value has been adopted in such a way that the additional noisy is about 1% of the average measured displacement data. The stopping criteria have been set by using Eq. (19) with $\epsilon^\star = 10^{-15}$ for the noiseless case. The stopping criteria have been set by using Eq. (22) with ϵ^2 defined by Eq. (21) for the noisy case.

If additional informations are available, for example the damage location and/or the damage dimension, they should be used in order to give a more suitable choice on the the initial guess for the conjugate gradient method. However, when these informations are not available, it has been observed that the initial guess of the unknown quantities, in general, can not be chosen arbitrarily [13]; i.e the conjugate gradient method could not be converge to the solution. This behaviour has been strongly noticed when a slightly high DOF dynamical system has been studied. For the case under concern the undamaged configuration has been adopted for the initial guess. Fig. 3 shows the damage factor, defined as

$$DF_i = \frac{K_i^u - K_i^d}{K_i^u} \quad i = 1, \ldots, N \qquad (26)$$

where K_i^u and K_i^d are the stiffness parameters for the undamaged and damaged structures, respectively.

It is important to observe that here the results are not only compliant with the real values of the considered damage configuration, but also some estimated stiffness parameters result greater than the reference values, as one can observe in the elements five and eleven, where the real structure is undamaged.

Figs. 4 and 5 present a comparison between the estimated and exact damage factor values when noiseless and noisy experimental data are used, respectively, by using the hybrid approach.

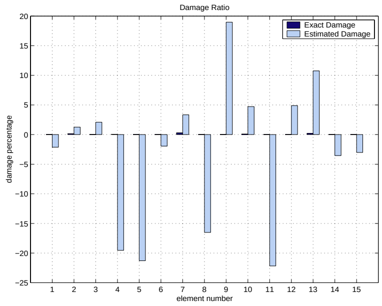

Figure 3: Estimated damage factor for a 3-bay truss structure when an arbitrary initial guess is employed.

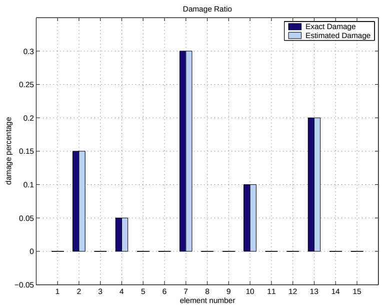

Figure 4: Estimated damage factor when the GA initial guess is employed (noiseless data).

Regarding the estimation results presented in Figs 4 and 5, it has been observed that the hybrid approach provides a perfect damage estimation when noiseless experimental data is employed. A more realistic case is considered when noisy experimental data is employed and even in this case satisfactory estimation of damage localization and quantification have been obtained, except for the bar number four. One possible explanation for the wrong estimation for the stiffness of this bar could be charged to the fact that the forces, that have been applied to the nodes III and IV (which correspond also to the nodes of the bar number four), do not

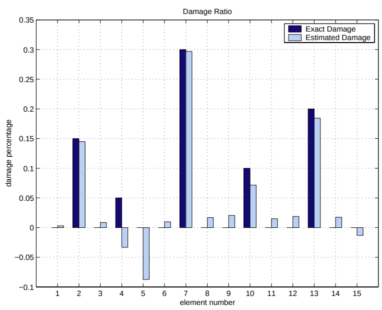

Figure 5: Estimated damage factor when the GA initial guess is employed (noisy data).

produced a sufficient elastic displacement on this element. So that no sufficient "informations" about the elastic behaviour of this element of the truss can be provided to the algorithm.

The Beam Structure

The numerical example considered here is a beam-like structure (see Fig. 2), modeled with 10 beam finite elements and clamped at one end. The aluminum beam ($\rho = 2700\ kg/m^3$ and $E = 70\ GPa$) present the following properties: rectangular cross section area $A = 4.5 \times 10^{-5}\ m^2$, length $L = 0.43\ m$ and inertial moment $I = 3.375 \times 10^{-11}$. As far as the damping matrix is concerned, it has been assumed that it is proportional to the undamaged stiffness matrix $\mathbf{C} = 10^{-3}\mathbf{K}$. In this case it has been assumed that the external force varies with time: $\mathbf{f}(t) = 5.0 - 2.0 \sin(\pi t)$ applied at node number 10, (see Fig. 2); with null initial conditions on both the displacement and the velocities of the nodes. Note that usually there is a difficulty on the experimental measuring of the rotational DOFs which must be used in the finite element representation. To overcome this problem, i.e the explicit dependence on the rotational DOFs in the finite element representation, the Guyan reduction has been employed in order to have a dependency only on the vertical displacements. The following damage configuration has been considered: a 20% damage over the element 2; 10% over the element 5; 15% over the element 9 and a 5% damage over the element 10. All the others elements have been assumed as undamaged.

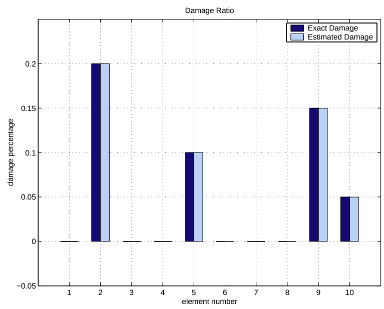

Figure 6: Estimated damage factor when the GA initial guess is employed (noiseless data).

As in the previous example two different cases have been considered: one with noiseless experimental data and one with noisy experimental data. Numerical simulations have been performed assuming the final time as $t_f = 2$ s and the time step as $\triangle t = 4.0 \times 10^{-4}$ s. Also in this case without the application of the hybrid approach it was not possible both to identify and localize the damage on the beam. By using the hybrid approach, the damage

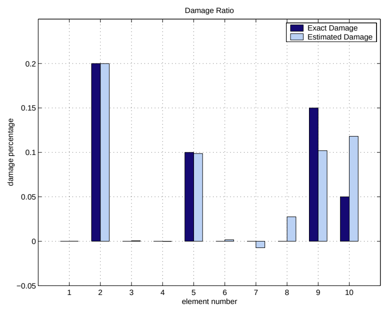

Figure 7: Estimated damage factor when the GA initial guess is employed (noisy data).

factor has been identified in each single beam element of the structure as shown in Figs. 6 and 7 for the noiseless and noisy cases, respectively.

Regarding the estimation results for the beam structure, once again it has been observed that, using the proposed hybrid approach, a perfect estimation has been achieved when noiseless experimental data is considered, while considering noisy experimental data a satisfactory damage estimation has been obtained.

FINAL REMARKS

The inverse vibration problem of estimating the unknown stiffness parameters (damage identification), of both a truss and a beam forced dynamic system has been solved using a hybrid approach, where it has been employed a stochastic scheme of minimization (GA) coupled to the conjugate gradient method.

The insertion of the genetic algorithm method into the computational procedure is justified because an arbitrary choice of the initial guess for the conjugate gradient method has been showed as an inappropriate procedure when dynamical systems with a high DOF number are considered.

The application of the hybrid approach has been required a higher CPU time, which could be seen as a drawback. In fact, as bigger is the dynamical system considered, more time-consuming is the computational procedure. However, this problem could be solved through a parallel implementation of the GA, since this kind of problem have already been solved for an inverse heat conduction problem [21].

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